

# Stochastic gravitational waves backgrounds: a probe for inflationary and non-inflationary cosmology<sup>a</sup>

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Physical scenarios, leading to highly energetic stochastic gravitational waves backgrounds (for frequencies ranging from the  $\mu\text{Hz}$  up to the  $\text{GHz}$ ) are examined. In some cases the typical amplitude of the logarithmic energy spectrum can be even eight orders of magnitude larger than the ordinary inflationary prediction. Scaling violations in the frequency dependence of the energy density of the produced gravitons are discussed.

## 1 Inflationary graviton spectra and their scaling properties

The fraction of critical energy density  $\rho_c$  stored in relic gravitons at the present (conformal) time  $\eta_0$  per each logarithmic interval of the physical frequency  $f$

$$\Omega_{\text{GW}}(f, \eta_0) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f} = \overline{\Omega}(\eta_0) \omega(f, \eta_0) \quad (1)$$

is the quantity we will be mostly interested in. The frequency dependence in  $\Omega_{\text{GW}}(f, \eta_0)$  is a specific feature of the mechanism responsible for the production of the gravitons and, in a given interval of the present frequency, the slope of the logarithmic energy spectrum can be defined as<sup>1</sup>

$$\alpha = \frac{d \ln \omega(f, \eta_0)}{d \ln f}. \quad (2)$$

If, in a given logarithmic interval of frequency,  $\alpha < 0$  the spectrum is *red* since its dominant energetical content is stored in the infra-red. If, on the other hand  $0 < \alpha \leq 1$  the spectrum is *blue*, namely a mildly increasing logarithmic energy density. Finally if  $\alpha > 1$  we will talk about *violet* spectrum whose dominant energetical content is stored in the ultra-violet. The case  $\alpha = 0$  corresponds to the case of scale-invariant (Harrison-Zeldovich) logarithmic energy spectrum.

Every variation of the background geometry produces graviton pairs which are stochastically distributed<sup>2</sup>. The amplitude of the detectable signal depends, however, upon the specific model of curvature evolution. In ordinary inflationary models the amount of gravitons produced by a variation of the geometry is notoriously quite small<sup>3</sup>. This feature can be traced back to the fact that  $\Omega_{\text{GW}}(f, \eta_0)$  is either a decreasing or (at most) a flat function of the present frequency. Suppose, for simplicity, that the ordinary inflationary phase is suddenly followed by a radiation dominated phase turning, after some time, into a matter dominated stage of expansion<sup>3</sup>. The logarithmic energy spectrum will have, as a function of the present

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frequency, two main branches : an infra-red branch (roughly ranging between  $10^{-18}$  Hz and  $10^{-16}$  Hz) and a flat (or possibly decreasing) branch between  $10^{-16}$  and 100 MHz.

The flat (or, more precisely, slightly decreasing) branch of the spectrum is mainly due to those modes leaving the horizon during the inflationary phase and re-entering during the radiation dominated epoch. The infra-red branch of the spectrum is produced by modes leaving the horizon during the inflationary phase and re-entering during the matter dominated epoch.

Starting from infra-red we have that the COBE observations of the first thirty multipole moments of the temperature fluctuations in the microwave sky imply that the GW contribution to the Sachs-Wolfe integral cannot exceed the amount of anisotropy directly detected. This implies that for frequencies  $f_0$  approximately comparable with  $H_0$  and  $20 H_0$  (where  $H_0$  is the present value of the Hubble constant including its indetermination  $h_0$ )  $h_0^2 \Omega_{\text{GW}}(f_0, \eta_0) < 7 \times 10^{-9}$ . Moving towards the ultra-violet, the very small size of the fractional timing error in the arrivals of the millisecond pulsar's pulses requires that  $\Omega_{\text{GW}}(f_P, \eta_0) < 10^{-8}$  for a typical frequency roughly comparable with the inverse of the observation time during which the pulses have been monitored, i.e.  $f_P \sim 10$  nHz.

Finally, if we believe the simplest (homogeneous and isotropic) big-bang nucleosynthesis (BBN) scenario we have to require that the total fraction of critical energy density stored in relic gravitons at the BBN time does not exceed the energy density stored in relativistic matter at the same epoch. Defining  $\Omega_\gamma(\eta_0)$  as the fraction of critical energy density presently stored in radiation we have that the BBN consistency requirement demands

$$h_0^2 \int_{f_{\text{ns}}}^{f_{\text{max}}} \Omega_{\text{GW}}(f, \eta_0) d \ln f < 0.2 h_0^2 \Omega_\gamma(\eta_0) \simeq 5 \times 10^{-6}, \quad (3)$$

where  $f_{\text{ns}} \simeq 0.1$  nHz is the present value of the frequency corresponding to the horizon at the nucleosynthesis time;  $f_{\text{max}}$  stands for the maximal frequency of the spectrum and it depends upon the specific theoretical model (in the case of ordinary inflationary models  $f_{\text{max}} = 100$  MHz). The constraint expressed in Eq. (3) is *global* in the sense that it bounds the *integral* of the logarithmic energy spectrum. The constraints coming from pulsar's timing errors and from the integrated Sachs-Wolfe effect are instead *local* in the sense that they bound the value of the logarithmic energy spectrum in a specific interval of frequencies.

In the case of stochastic GW backgrounds of inflationary origin, owing to the red nature of the logarithmic energy spectrum, the most significant constraints are the ones present in the soft region of the spectrum, more specifically, the ones connected with the Sachs-Wolfe effect. Taking into account the specific frequency behavior in the infra-red branch of the spectrum and assuming perfect scale invariance we have that  $h_0^2 \Omega_{\text{GW}}(f, \eta_0) < 10^{-15}$  for frequencies  $f > 10^{-16}$  Hz. We have to conclude that the inflationary spectra are invisible by pairs of interferometric detectors operating in a window ranging approximately between few Hz and 10 kHz.

In order to illustrate more quantitatively this point we remind the expression of the signal-to-noise ratio (SNR) in the context of optimal processing required for

the detection of stochastic backgrounds<sup>4</sup>. By assuming that the intrinsic noises of the detectors are stationary, gaussian, uncorrelated, much larger in amplitude than the gravitational strain, and statistically independent on the strain itself, one has:

$$\text{SNR}^2 = \frac{3H_0^2}{2\sqrt{2}\pi^2} F \sqrt{T} \left\{ \int_0^\infty df \frac{\gamma^2(f) \Omega_{\text{GW}}^2(f)}{f^6 S_n^{(1)}(f) S_n^{(2)}(f)} \right\}^{1/2}, \quad (4)$$

( $F$  depends upon the geometry of the two detectors and in the case of the correlation between two interferometers  $F = 2/5$ ;  $T$  is the observation time). In Eq. (4),  $S_n^{(k)}(f)$  is the (one-sided) noise power spectrum (NPS) of the  $k$ -th ( $k = 1, 2$ ) detector. The NPS contains the important informations concerning the noise sources (in broad terms seismic, thermal and shot noises) while  $\gamma(f)$  is the overlap reduction function which is determined by the relative locations and orientations of the two detectors. Without going through the technical details<sup>5</sup> from the expression of the SNR we want to notice that the achievable sensitivity of a pair of wide band interferometers crucially depends upon the spectral slope of the theoretical energy spectrum in the operating window of the detectors. So, a flat spectrum will lead to an experimental sensitivity which might not be similar to the sensitivity achievable in the case of a blue or violet spectra<sup>5,7</sup>. In the case of an exactly scale invariant spectrum the correlation of the two (coaligned) LIGO detectors with central corner stations in Livingston (Louisiana) and in Hanford (Washington) will have a sensitivity to a flat spectrum which is  $h_0^2 \Omega_{\text{GW}}(100 \text{ Hz}) \simeq 6.5 \times 10^{-11}$  after one year of observation and with signal-to-noise ratio equal to one<sup>5</sup>. This implies that ordinary inflationary spectra are (and will be) invisible by wide band detectors since the inflationary prediction, in the most favorable case (i.e. scale invariant spectra), undershoots the experimental sensitivity by more than four orders of magnitude.

## 2 Scaling violations in graviton spectra

In order to have a large detectable signal between 1 Hz and 10 kHz we have to look for models exhibiting scaling violations for frequencies larger than the mHz. The scaling violations should go in the direction of blue ( $0 < \alpha \leq 1$ ) or violet ( $\alpha > 1$ ) logarithmic energy spectra. Only in this case we shall have the hope that the signal will be large enough in the window of wide band detectors. Notice that the growth of the spectra should not necessarily be monotonic: we might have a blue or violet spectrum for a limited interval of frequencies with a spike or a hump.

### 2.1 Quintessential inflationary models

Suppose now, as a toy example, that the ordinary inflationary phase is not immediately followed by a radiation dominated phase but by a quite long phase expanding slower than radiation<sup>1</sup>. This speculation is theoretically plausible since we ignore what was the thermodynamical history of the Universe prior to BBN. If the Universe expanded slower than radiation the equation of state of the effective sources driving the geometry had to be, for some time, stiffer than radiation. This means that the effective speed of sound  $c_s$  had to lie in the range  $1/\sqrt{3} < c_s \leq 1$ . Then

the resulting logarithmic energy spectrum, for the modes leaving the horizon during the inflationary phase and re-entering during the stiff phase, is tilted towards large frequencies with typical (blue) slope given by<sup>1</sup>

$$\alpha = \frac{6c_s^2 - 2}{3c_s^2 + 1}, \quad 0 < \alpha \leq 1. \quad (5)$$

A situation very similar to the one we just described occurs in quintessential inflationary models<sup>6</sup>. In this case the tilt is maximal (i.e.,  $\alpha = 1$ ) and a more precise calculation shows the appearance of logarithmic corrections in the logarithmic energy spectrum which becomes<sup>1,6,7</sup>  $\omega(f) \propto f \ln^2 f$ . The maximal frequency  $f_{\max}(\eta_0)$  is of the order of 100 GHz (to be compared with the 100 MHz of ordinary inflationary models) and it corresponds to the typical frequency of a spike in the GW background. In quintessential inflationary models the relic graviton background will then have the usual infra-red and flat branches supplemented, at high frequencies (larger than the mHz and smaller than the GHz) by a true hard branch<sup>7</sup> whose peak can be, in terms of  $h_0^2 \Omega_{\text{GW}}$ , of the order of  $10^{-6}$ , compatible with the BBN bound and roughly eight orders of magnitude larger than the signal provided by ordinary inflationary models.

An interesting aspect of this class of models is that the maximal signal occurs in a frequency region between the MHz and the GHz. Microwave cavities can be used as GW detectors precisely in the mentioned frequency range<sup>9</sup>. There were published results reporting the construction of this type of detectors<sup>10</sup> and the possibility of further improvements in the sensitivity received recently attention<sup>11</sup>. Our signal is certainly a candidate for this type of devices.

## 2.2 String cosmological models

In string cosmological models<sup>12</sup> of pre-big-bang type  $h_0^2 \Omega_{\text{GW}}$  can be as large as  $10^{-7}$ – $10^{-6}$  for frequencies ranging between 1 Hz and 100 GHz<sup>3,8</sup>. In these types of models the logarithmic energy spectrum can be either blue or violet depending upon the given mode of the spectrum. If the mode under consideration left the horizon during the dilaton-dominated epoch the typical slope will be violet (i.e.  $\alpha \sim 3$  up to logarithmic corrections). If the given mode left the horizon during the stringy phase the slope can be also blue with typical spectral slope  $\alpha \sim 6 - 2(\ln g_1/g_s / \ln z_s)$  where  $g_1$  and  $g_s$  are the values of the dilaton coupling at the end of the stringy phase and at the end of the dilaton dominated phase;  $z_s$  parametrizes the duration of the stringy phase. This behaviour is representative of the minimal string cosmological scenarios. However, in the non-minimal case the spectra can also be non monotonic<sup>8</sup>. Recently the sensitivity of a pair of VIRGO detectors to string cosmological gravitons was specifically analyzed<sup>13</sup> with the conclusion that a VIRGO pair, in its upgraded stage, will certainly be able to probe wide regions of the parameter space of these models. If we maximize the overlap between the two detectors<sup>13</sup> or if we would reduce (selectively) the pendulum and pendulum's internal modes contribution to the thermal noise of the instruments<sup>14</sup>, the visible region (after one year of observation and with SNR equal to one) of the parameter space will get even larger. Unfortunately, as in the case of the advanced LIGO detectors, also in

the case of the advanced VIRGO detector the sensitivity to a flat spectrum will be irrelevant for ordinary inflationary models. Finally, it is worth mentioning that blue and violet logarithmic energy spectra can also arise in the context of other models like dimensional decoupling<sup>15</sup> and early violations of the dominant energy condition<sup>16</sup> in Einsteinian theories of gravity.

### 3 Relic gravitons from local processes inside the horizon

GW can be produced not only because of the adiabatic variation of the background geometry, but also because there are physical processes occurring inside the horizon producing large amounts of gravitational radiation. Typical examples of such a statement are topological defects models, strongly first order phase transitions (where the bubble collisions can produce spikes in the GW background for frequencies roughly comparable with the inverse of horizon/bubble size). For instance, if the EWPT would be strongly first order we would have spikes in the graviton background for frequencies between 10  $\mu$ Hz and 0.1 mHz. In the following we want to discuss a further mechanism connected with the existence of hypermagnetic fields in the symmetric phase of the electroweak theory<sup>17</sup>.

#### 3.1 Magnetic and Hypermagnetic Knots

Since a generic magnetic field configuration at finite conductivity leads to an energy-momentum tensor which is anisotropic and which has non-vanishing transverse and traceless component (TT), if magnetic fields are present inside the horizon at some epoch they can radiate gravitationally. The TT components of the energy momentum tensor act as a source term for the TT fluctuations of the geometry which are associated with gravitational waves. A non-trivial example of this effect is provided by magnetic knot configurations<sup>18</sup> which are transverse (magnetic) field configurations with a topologically non-trivial structure in the flux lines. These configurations can also be generated by direct projection of a pure  $SU(2)$  field onto a fixed (electromagnetic) direction in isospace<sup>19</sup>. In magnetohydrodynamics (MHD) magnetic knots configurations are stable and conserved by plasma evolution provided the conductivity is sufficiently large. The degree of knottedness of the configuration is measured by the magnetic helicity. Assuming a specific configuration<sup>18</sup> the frequency of the hump in the GW spectrum could range between  $10^{-14}$  Hz and  $10^{-12}$  Hz.

For sufficiently high temperatures and for sufficiently large values of the various fermionic charges the  $SU(2)_L \otimes U(1)_Y$  symmetry is restored and, thence, non-screened vector modes will now correspond to the hypercharge group. Topologically non-trivial configurations of the hypermagnetic field ( $\vec{\mathcal{H}}_Y$ ) can be related to the baryon asymmetry of the Universe (BAU)<sup>17,20</sup> and they can also radiate gravitationally<sup>21,22</sup>. The evolution equations of the hypercharge field at finite conductivity imply that the largest modes which can survive in the plasma are the ones associated with the hypermagnetic conductivity frequency which is roughly eight orders of magnitude smaller than the temperature at the time of the electroweak phase transition which I take to occur around 100 GeV. The logarithmic energy spectra

of the produced gravitons can be different depending upon the specific form of the configuration. However, we can estimate

$$h_0^2 \Omega_{\text{GW}}(f, \eta_0) \simeq 10^{-6} \delta^4, \quad (6)$$

where  $\delta = |\vec{\mathcal{H}}_Y|/T_{\text{ew}}^2$  and  $T_{\text{ew}}$  roughly corresponds to 100 GeV. Notice that  $\delta \sim 1$  does not violate the closure density bound since it could be divided by  $N_{\text{eff}}$  (i.e. the effective number of spin degrees of freedom at  $T_{\text{ew}}$ ) which is already included in the numerical prefactor of our estimate. The frequency  $f$  lies in the range between 10  $\mu\text{Hz}$  and the few kHz. The lower frequency corresponds to the frequency of the horizon at the electroweak epoch, i.e.

$$f_{\text{ew}}(\eta_0) \sim 0.201 \left( \frac{T_{\text{ew}}}{1 \text{ GeV}} \right) \left( \frac{N_{\text{eff}}}{100} \right)^{1/6} \mu\text{Hz}. \quad (7)$$

The higher frequency roughly corresponds to the hypermagnetic conductivity frequency, namely  $f_\sigma(\eta_0) \sim 10^8 f_{\text{ew}}(\eta_0)$ . The presence of a classical hypermagnetic background in the symmetric phase of the electroweak theory produces interesting non linear effects in the phase diagram of the electroweak phase transition<sup>20</sup>. If we then suppose<sup>20</sup> that  $\delta > 0.3$  we can get  $h_0^2 \Omega_{\text{GW}}$  as large as  $10^{-7}$ . This signal satisfies the above mentioned phenomenological bounds on the graviton backgrounds of primordial origin<sup>21</sup>.

#### 4 Final remarks

In spite of the fact that ordinary inflationary models provide a rather minute relic graviton background, we showed that various physical situations can provide a much larger signal. A pair of correlated and coaligned VIRGO detectors (for  $\text{Hz} < f < 10 \text{ kHz}$ ) and microwave cavities (for  $\text{MHz} < f < \text{GHz}$ ) can offer exciting detectability prospects.

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